Fission Cross Section of Pu^{241}†

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The low-energy neutron-induced fission cross section of Pu^{241} has been measured from 2–100 eV, with sufficiently high resolution to permit multilevel analysis below neutron energies of 36 eV. The results of the analysis are consistent with the assumption that the resonances can be separated into two groups having different average fission widths, and that the interference effects can be described by a single-fission channel for each group. The properties of resonances from 12–36 eV are found to be very similar to those below 12 eV, described previously. On the basis of the parameters obtained, the number of effective fission channels is discussed.

I. INTRODUCTION

PREVIOUS measurements of the total neutron cross section of Pu^{241} by Simpson and Schuman[1] permitted a multilevel analysis for the resonances lying below 12 eV, carried out by Simpson and Moore.[2] This analysis indicated that the resonances in Pu^{241} can be separated into two groups with different average fission widths. Interference in fission is observed among the wide levels of one group and among the narrow levels of the other group, but not between members of different groups.

The analysis of Simpson and Moore[2] also described the fission cross section of Pu^{241}, which was subsequently measured by Watanabe and Simpson.[3] The resolution of the latter measurement was poorer than that of the total cross-section measurement[1] and did not permit an extension of the analysis to higher energies. In the present measurement of the fission cross section of Pu^{241}, the resolution was high enough to permit level parameters to be obtained and a multilevel analysis to be carried out for neutron energies below 36 eV.

II. EXPERIMENTAL MEASUREMENT

The experimental data were obtained with the use of the Rensselaer Polytechnic Institute (RPI) linear electron accelerator.[4] Bursts of approximately 50-MeV electrons from the accelerator were allowed to strike a water-cooled tungsten target. The tungsten target served both as an electron-photon converter and as a photoneutron source, and the cooling water served as a neutron moderator. Neutrons from the target were collimated as shown schematically in Fig. 1, and allowed to strike the fission detector located at a distance of 10.56 m from the target. The energies of the neutrons were determined by measuring their time-of-flight.

The fission detector, also shown in Fig. 1, consisted of six independent gas-scintillation chambers, which used an argon-nitrogen mixture as the scintillating gas. The fission detector contained six 5-mil nickel foils, each $2\frac{1}{2}$ in. in diameter, on which were deposited a total of 67 mg of plutonium in the form of PuF_3. Four of the fission foils, with 42 mg of plutonium, consisted of 93.3% Pu^{241} and 1.1% Pu^{239}, which were the only fissile isotopes. The remaining two foils consisted of 25 mg of Pu^{239}, to permit corrections for the Pu^{239} contaminant in the Pu^{241}. (These corrections were subsequently found to be negligible.) The foils were positioned back-to-back, such that all were at the same neutron flight path.

The fission detector was designed to minimize scattering effects by placing only a small amount of material in the neutron beam. In addition to the plutonium and nickel backings, the only material in the collimated beam was a total of 0.030 in. of Al, which formed the chamber end windows, and two boron trifluoride proportional tubes, 1 in. in diameter.

The two BF_3 counters, depleted to approximately 11% B^{10}, served as a flux monitor. They were effectively located at the same flight path as the fission foils. This was accomplished by placing one BF_3 tube just in front of the flight path at which the plutonium foils were located (not, however, in the same part of the beam) and one just behind. This arrangement is also shown in Fig. 1. The effective thickness (2σ) of the BF_3 detector system was 2.30 in. It was assumed that the efficiency of the BF_3 system varies inversely as the neutron velocity at neutron energies from 2–100 eV.

The gamma flash affected both the fission and BF_3 detector systems, tending to paralyze both systems for perhaps 5–10 μsec after the burst. Complete recovery from this paralysis may have taken as long as 50 μsec

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[1] O. D. Simpson and R. P. Schuman, Nucl. Sci. Eng. **11**, 111 (1961).
[2] O. D. Simpson and M. S. Moore, Phys. Rev. **123**, 559 (1961).
[3] T. Watanabe and O. D. Simpson, Phys. Rev. **133**, B390 (1964).
[4] E. R. Gaerttner, M. L. Yeater, and R. R. Fullwood, in *Neutron Physics*, edited by M. L. Yeater (Academic Press Inc., New York, 1962), p. 263.

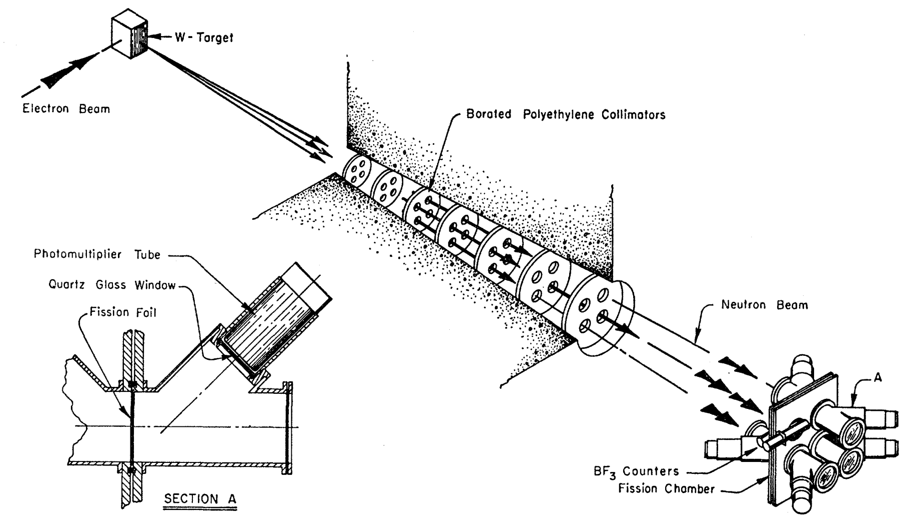

FIG. 1. Experimental arrangement, showing fission chambers and $B^{10}F_3$ counters in neutron beam produced by the accelerator. Collimation system used is indicated schematically.

(corresponding to a neutron energy of approximately 250 eV).

It was noted that the neutron spectrum from the tungsten target contained a significant Maxwellian (thermal-neutron) component. This necessitated the interposing of a 30-mil cadmium plate in the beam to remove overlap neutrons (neutrons produced in one burst but counted as though they had been produced in a subsequent burst because their flight times are longer than the time between bursts). The highest accelerator repetition rate used was 360 bursts/sec, so the filter was very effective in eliminating overlap neutrons. Some distortion of the Pu^{241} cross section due to resolution differences between the two detector systems might be expected in the vicinity of higher energy cadmium resonances, since the BF_3 detector thickness (2.3 in.) was not negligible. However, it was found that the Pu^{241} cross section was very low in the vicinity of all Cd resonances between 2 and 60 eV, so that no resolution correction was required for this effect in the region where the resonance analysis was carried out.

The fission-fragment pulse-height spectrum was good enough that virtually all the background due to natural radioactivity of the fissile material was eliminated by a voltage bias setting. The observed backgrounds could be attributed primarily to fast neutrons scattered by the detector, moderated in the surroundings and re-entering the detector to be counted with higher efficiency.[5] The time-of-flight spectrum of this background was measured by interposing into the neutron beam a borated polyethylene filter, which effectively removed all neutrons below 100 eV but permitted about 50% of the fast neutrons to pass through. The time-of-flight spectrum of background neutrons for the BF_3 detector was found to have the same shape as the open beam spectrum, so that a highly precise determination of the background level was not necessary to the present (relative) measurement. Normalization of the fission background was done by attributing the observed fission counting rate to background at neutron energies at which the fission cross section is known to be very low (2 and 19.5 eV).[1,3] The backgrounds were high; the background counting rate amounted to approximately 10% of the observed rate even over the largest peaks, and the statistical accuracy of the data was correspondingly reduced.

The experimental data are shown in Figs. 2–6. Normalization of the fission cross section was done, following the method of Watanabe and Simpson,[3] by requiring the fission cross section at the peak of the

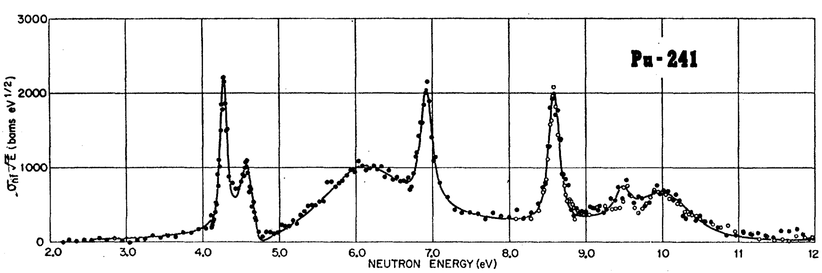

FIG. 2. The fission cross section of Pu^{241}, multiplied by the square root of the neutron energy, from 2 to 12 eV. Solid and open circles represent data taken with time-of-flight analyzer channel widths of $\frac{1}{2}$ and $\frac{1}{4}$ μsec, respectively. The solid curve represents the results of multilevel analysis of these data.

[5] O. D. Simpson, R. G. Fluharty, M. S. Moore, N. H. Marshall, F. B. Simpson, G. E. Stokes, T. Watanabe, and T. E. Young, Nucl. Instr. Methods (to be published).

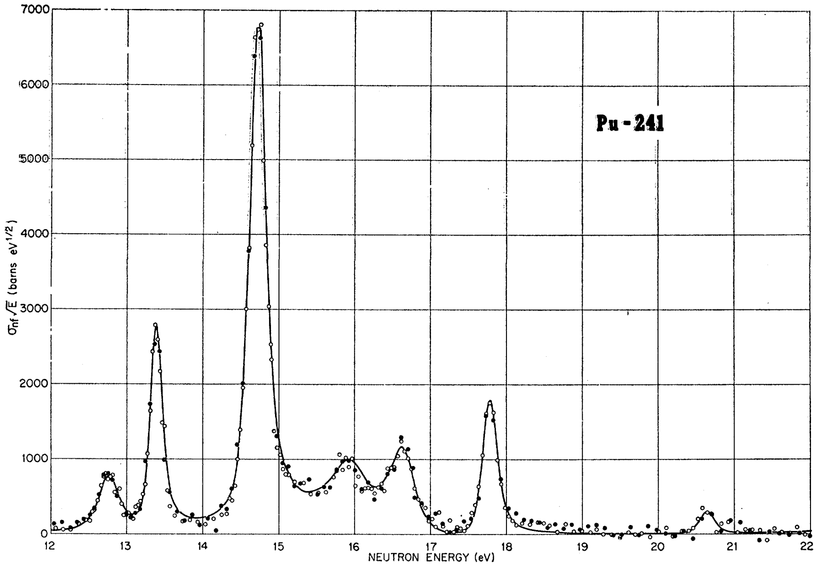

FIG. 3. The fission cross section of Pu^{241}, multiplied by the square root of the neutron energy, from 12 to 22 eV. Solid and open circles represent data taken with time-of-flight analyzer channel widths of $\frac{1}{2}$ and $\frac{1}{4}$ μsec, respectively. The solid curve represents the results of multilevel analysis of these data.

6-eV resonance to be 406 b, based on the analysis[2] of the total cross section of Pu^{241}. Normalization of the two independent runs shown in Figs. 2–6 was done by equating the integrated cross section from 8–100 eV for the two runs.

III. RESOLUTION AND DOPPLER EFFECTS

Multilevel analysis is primarily shape analysis, which depends strongly on the shape of the wings of the resonances. If the wings of the resonances are obscured by resolution effects and by close-lying weak levels, multilevel analysis is not a fruitful approach. In order to determine over how large a region of neutron energies such an analysis might be useful, the resolution and Doppler-broadening effects must be considered. It was assumed that the Doppler broadening introduced by the thermal motion of the fissile target nuclei could be adequately treated from 2–36 eV by a gas-model approximation.[6] The plutonium on the fission foils was in the chemical form of PuF_3, and the Debye temperature correction was neglected. Resolution broadening can arise from two sources: time-of-flight uncertainties and flight-path uncertainties. Time-of-flight uncertainties are due to the finite burst width (0.3 μsec), the channel width of the time-of-flight analyzer (0.25 or 0.5 μsec for the different runs shown in Figs. 2–6), and the incoherence between the burst and the time-of-flight analyzer crystal oscillator (0.25 μsec). Flight-path uncertainties are due to the source and detector thicknesses. The width in energy of the broadening function due to flight-path uncertainties varies as the neutron energy E_n; that due to time-of-flight uncertainties varies as $E_n^{3/2}$. The width of the Doppler-broadening function varies as $E_n^{1/2}$. The variation with E_n of the calculated Doppler and time-of-flight resolution functions is shown in Fig. 7.

In order to obtain an estimate of the flight-path uncertainty, which was not readily calculable, the following procedure was used: A single-level Breit-Wigner calculation was made of the fission cross section of several Pu^{239} resonances, whose parameters had been obtained by Bollinger *et al.*[7] Next, a numerical convolution was performed[8] of the calculated Pu^{239} cross section with the Doppler and time-resolution-broadening func-

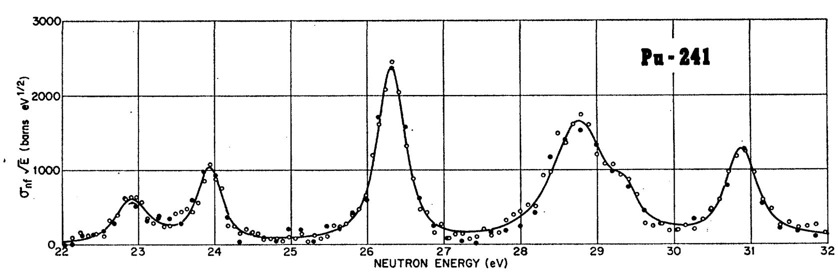

FIG. 4. The fission cross section of Pu^{241}, multiplied by the square root of the neutron energy, from 22 to 32 eV. Solid and open circles represent data taken with time-of-flight analyzer channel widths of $\frac{1}{2}$ and $\frac{1}{4}$ μsec, respectively. The solid curve represents the results of multilevel analysis of these data.

[6] W. E. Lamb, Jr., Phys. Rev. **55**, 190 (1939).
[7] L. M. Bollinger, R. E. Coté, and G. E. Thomas, Proc. Intern. Conf. Peaceful Uses At. Energy, Geneva **15**, 127 (1958).
[8] N. H. Marshall and O. D. Simpson, U. S. Atomic Energy Commission Report No. IDO-16954, 1963 (unpublished).

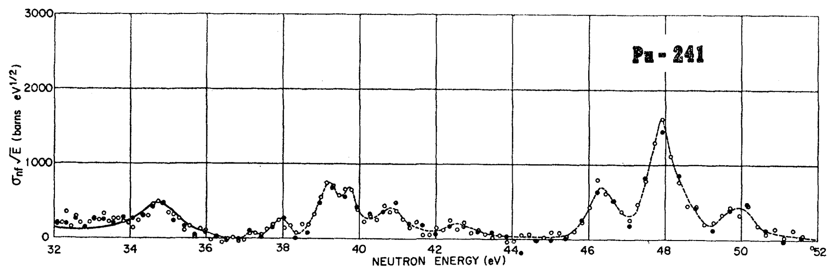

FIG. 5. The fission cross section of Pu^{241}, multiplied by the square root of the neutron energy, from 32 to 52 eV. Solid and open circles represent data taken with time-of-flight analyzer channel widths of $\frac{1}{2}$ and $\frac{1}{4}$ μsec, respectively. The solid curve represents the results of multilevel analysis of these data, which was carried out below 36 eV. Above 36 eV, the resolution did not permit such an analysis; the dashed line indicates the prominent structure observed.

tions (Fig. 7). The results, compared with the experimentally measured Pu^{239} cross section, showed that the functional form of the Doppler and time-of-flight resolution was adequate to describe the observed resonance broadening, and that flight-path uncertainties could be neglected.

From Fig. 7, it can be seen that the Doppler function creates most of the resonance broadening below 20 eV. The resolution function becomes prohibitively large above approximately 40 eV, at least for the narrow resonances. At low energies, where Doppler broadening is dominant, there is no appreciable difference in the width or peaks of the resonances as measured with channel widths of 0.25 μsec or with 0.5 μsec, as seen in Figs. 2–6. At higher energies some resolution differences between the two runs can be discerned. In order to determine the range of neutron energies over which the multilevel approach may be used, a plot (Fig. 8) was made showing the number of levels having a characteristic energy less than the neutron energy E_n as a function of E_n. The integral distribution begins to show appreciable curvature in the region of neutron energies above 40 eV, showing that levels are being obscured by resolution broadening. Consequently, the analysis was carried out only for resonances below 36 eV, and in particular for those resonances indicated by the dashed line in Fig. 8.

IV. MULTILEVEL ANALYSIS

Preliminary estimates of the fission widths for the resonances below 36 eV were obtained by comparing these fission data with total cross-section data, obtained previously by Simpson and Schuman,[1,9] which had almost exactly the same resolution. If it can be assumed that the radiative capture width Γ_γ is constant (in this case Γ_γ was assumed to be equal to 0.040 eV), a reasonable preliminary value for the fission width can be obtained by comparing fission and total cross-section data with equivalent resolutions.

The multilevel analysis was carried out under the assumption made by Simpson and Moore[2] for the analysis of the Pu^{241} total cross section below 12 eV. These assumptions may be summarized as follows: There are two groups of resonances. Members of different groups do not interfere with one another. Members of the same group interfere completely, in a single-fission channel. The same assumptions were found to be adequate for the present analysis. Adjustments of the preliminary parameters were made by trial-and-error, comparing the multilevel-predicted cross sections (with appropriate Doppler and resolution broadening) to the data until a consistent set of parameters was obtained. The description of the data given by the multilevel analysis is shown as the solid curve in Figs. 2–5, and may be considered to be adequate. The multilevel parameters are listed in Table I.

It should be pointed out that even under the most restrictive assumption possible, that of a single-fission channel in each of the two groups of levels, a unique assignment of resonances to one or the other group is not possible. In certain energy regions the interference effects are vanishingly small, so that certain resonances or energy regions are effectively independent of effects outside these regions. For example, from 2–12 eV, each of the observed resonances can be assigned to either group "A" or group "B." Again, between 12–20 eV, an assignment of each resonance can be made to either group A or group B. However, since these two regions

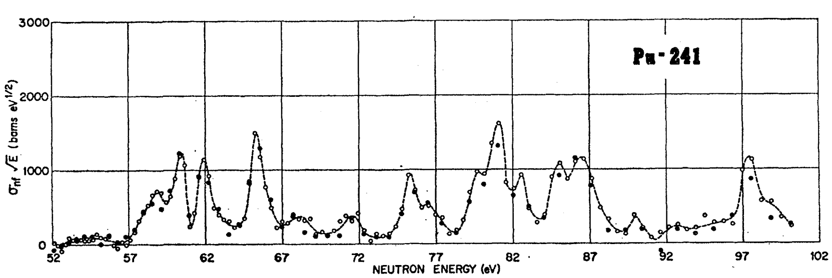

FIG. 6. The fission cross section of Pu^{241}, multiplied by the square root of the neutron energy, from 52 to 100 eV. Solid and open circles represent data taken with time-of-flight analyzer channel widths of $\frac{1}{2}$ and $\frac{1}{4}$ μsec, respectively. Above 36 eV, the resolution did not permit multilevel analysis; the dashed line indicates the prominent structure observed.

[9] O. D. Simpson and N. H. Marshall, U. S. Atomic Energy Commission Report No. IDO-16679, 1961 (unpublished).

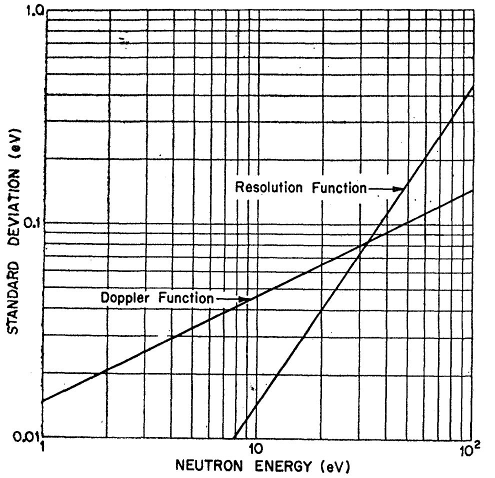

FIG. 7. Variation of experimental Doppler- and resolution-broadening widths, as a function of neutron energy. Plotted are values of the standard deviation in eV for a functional form which is constant in the appropriate coordinate system (velocity for the Doppler, and time-of-flight for the resolution function).

are virtually independent, it is not clear that group A from 2–12 eV is the same as group A from 12–20 eV. Such uncertainties are shown by dashed lines in Table I.

TABLE I. Multilevel parameters for resonances in Pu^{241}. These parameters correspond to the fit to the data shown by the solid curve in Figs. 2–5. The fission width $\Gamma_{\lambda f}$ appears in two columns, labeled group A or group B as is appropriate, as determined from a two-fission-channel analysis. In the analysis, $\Gamma_{\lambda\gamma}$ is assumed to have the value 40×10^{-3} eV, and g was assumed to be $\frac{1}{2}$ for all resonances. Although a two-fission channel, one-spin-state analysis was carried out, the results hold equally well for two-spin states with a single-fission channel in each. Relative signs of $(\Gamma_{\lambda n}\Gamma_{\lambda f})^{1/2}$ are those which correctly describe the interference among fission resonances. The significance of dashed lines separating various sections of the table is explained in the text.

Level (λ)	E_λ (eV)	$2g\Gamma_\lambda^0 n$ (10^{-3} eV)	$\Gamma_{\lambda f}$ (10^{-3} eV) Group A	$\Gamma_{\lambda f}$ (10^{-3} eV) Group B	$\Gamma_{\lambda\gamma}$ (10^{-3} eV)	Relative signs of $(\Gamma_{\lambda n}\Gamma_{\lambda f})$
1	4.275	0.404	0	21	40	+
2	4.580	0.204	140	0	40	−
3	5.910	1.020	1350	0	40	−
4	6.915	0.275	0	93	40	−
5	8.585	0.324	0	70	40	+
6	9.48	0.068	0	125	40	−
7	10.11	0.47	900	0	40	+
8	12.77	0.22	250	0	40	−
9	13.38	0.5	0	50	40	−
10	14.73	1.61	135	0	40	+
11	16.01	0.36	500	0	40	−
12	16.65	0.36	0	300	40	+
13	17.78	0.41	0	80	40	+
14	20.63	0.08	0	40	40	−
15	22.86	0.24	0	400	40	+
16	23.96	0.31	0	230	40	−
17	26.34	0.82	0	280	40	+
18	28.75	1.12	750	0	40	+
19	29.35	0.10	0	40	40	−
20	30.88	0.45	0	300	40	+
21	34.90	0.45	1200	0	40	−

Since the average fission width of the two groups in each energy region is markedly different, it can perhaps be argued with some justification that the separation of levels into the particular groups shown in Table I is significant, even though the results of the analysis do not indicate which levels belong to which group.

It will be noted that the parameters listed in Table I for several of the resonances below 12 eV are different from those reported by Simpson and Moore.[2] The analysis by Simpson and Moore was carried out for the Pu^{241} total cross-section data of Simpson and Schuman.[1] More recent measurements of the total cross section of Pu^{241} by Pattenden[10] and by Craig and Westcott[11] are much more nearly consistent with the parameters of Table I than are the older data of Simpson and Schuman. (In the older data, the sample was undoubtedly too thick to give accurate values for the narrow resonance peaks.)

The data of Pattenden also show corroborating evidence for small levels whose existence was doubted in the present treatment and which were neglected in the analysis. These levels are at approximately 21, 28, and 33 eV. (These resonances are included in Fig. 8, as shown by the solid line.) Including them in the analysis could significantly improve the fit to the data in these energy regions.

V. POSSIBLE IMPROVEMENTS IN THE ANALYSIS

The target nucleus Pu^{241} is even-odd, so resonances in the compound nucleus Pu^{241} are characteristic of one of two possible spins. If it is assumed that the assignment of levels into two groups is valid, then the obvious interpretation of the two groups is that the resonances belonging to a given group are characteristic of the same compound nucleus spin, and that the average fission width of the resonances is also spin-dependent.

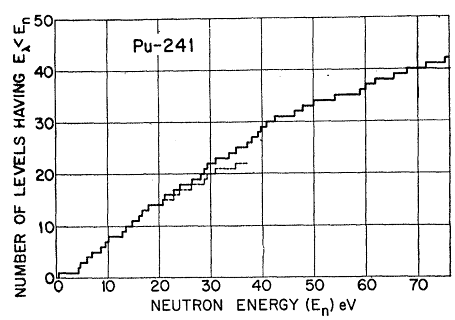

FIG. 8. Number of levels having $E_\lambda < E_n$ as a function of neutron energy E_n. The distribution indicates that an appreciable number of levels are being missed above 40 eV.

[10] N. J. Pattenden and S. Bardsley, Proceedings of the International Conference on Neutron Physics with Reactor Neutrons, U. S. Atomic Energy Commission Report No. ANL-6797, 1963 edited by F. E. Throw, p. 369 (unpublished).

[11] D. S. Craig and C. H. Westcott, Bull. Am. Phys. Soc. 7, 305 (1962); and (private communication).

The parameters given in Table I may be used to give some information about the number of effective fission channels. If it is assumed that the grouping of levels is correctly given in Table I, the distribution of fission widths can be used to calculate the number of effective fission channels, by the method of Wilets.[12] (See Sec. VI for a more complete discussion.) Carrying out this calculation for the resonances below 20 eV, where resolution and Doppler effects are small, one finds that there should be 3.0 ± 0.6 effective fission channels for group A and 3.1 ± 0.9 for group B. It appears on this basis that the assumption of but a single-fission channel for each group is inconsistent and much too restrictive.

Consequently, an attempt was made to improve the fit to the data under the assumption of more than one channel in each group. Particular energy regions considered were the valley near 5 eV, the doublet of resonances near 10 eV, and the triplet at 14–16 eV. In all cases it was found that the partial fission width vectors for close-lying resonances are very nearly parallel or antiparallel (complete interference in a single-fission channel) or very nearly orthogonal (no interference). Some improvements were made in the fit to the data by this approach; however, the results of the analysis are much more ambiguous in that a clear assignment of resonances to different groups is no longer possible.

It cannot be emphasized too strongly that the technique of multilevel analysis of cross-section data alone does not and cannot yield unique parameters. The admission of more than one fission channel per group almost precludes any assignment of resonances to different groups. In order to retain any hope of uniqueness, additional data are required: A measurement of the resonance spins or of some quantity characteristic of spin or of the fission channel involved.

VI. DISCUSSION AND CONCLUSIONS

A. Current Ideas About Fission

The resonance structure observed when slow neutrons are allowed to interact with a heavy target nuclide such as an isotope of uranium or plutonium appears to be most adequately treated under the assumption that the reduced widths for such resonances have a statistical behavior. It was proposed by Porter and Thomas[13] that the distribution of level widths of such a compound nucleus should follow a chi-squared distribution, with the number of degrees of freedom of the chi-squared distribution corresponding to the number of channels, or modes of decay of the compound nucleus, for the process under consideration. It has been reasonably well established that the reduced widths for elastic scattering by resonances of the same spin and parity in the compound nucleus follow a chi-squared distribution with one degree of freedom.[13,14] The radiative capture widths of such resonances are found to be reasonably constant from level to level, corresponding to a chi-squared distribution with a large number of degrees of freedom. Each of the many possible gamma-ray transitions from the given resonance level is expected to correspond to one of the many channels for the radiative process. Indeed, it has been shown that if one considers a single gamma-ray transition, interference effects are observed,[15] and the distribution of partial radiative widths for such a transition is consistent with a chi-squared distribution for 1 deg of freedom.[16]

The fission widths appear to show an intermediate behavior. Although the fission widths do not seem to vary as widely as the neutron widths, they do show much more variation than the radiative capture widths. The fission width distribution usually resembles a chi-squared distribution with perhaps 2 to 4 deg of freedom.

Bohr[17] has proposed a model for fission which provides a reasonable description of fission as a few-channel process, by considering the properties of the compound nucleus near the saddle point in the potential barrier which leads to fission. According to the Bohr model, the spectrum of quantum states energetically available to the fissioning nucleus at the saddle point determines the number of fission channels which are reflected by the fission width distribution. Bohr's description of fission as a few-channel process depends on the assumption that the fissioning nucleus at the saddle point is highly deformed. If most of the energy of the system is potential energy of deformation, and the nucleus is cold, i.e., not highly excited, then only a relatively few quantum states or channels are available to such a system. Since the system is highly deformed, these quantum states should be similar to those near the ground state of deformed nuclei, representing the relatively simple modes of motion, collective vibrations and rotations, which characterize the low-lying levels of such deformed nuclei. The ideas of Bohr concerning the channel structure of fissioning nuclides have been useful in analyzing measurements of the angular distribution of fragments of fission induced by fast neutron bombardment,[18,19] and by (d,p) reactions.[20–22]

[12] L. Wilets, Phys. Rev. Letters **9**, 430 (1962).
[13] C. E. Porter and R. G. Thomas, Phys. Rev. **104**, 483 (1956).
[14] J. D. Garrison, Proceedings of the Symposium on Statistical Properties of Atomic and Nuclear Spectra, Stony Brook, New York, edited by P. B. Kahn, 1963 (unpublished), and U. S. Atomic Energy Commission Report No. BNL-7402, 1964 (unpublished).
[15] R. E. Coté and L. M. Bollinger, Phys. Rev. Letters **6**, 695 (1961).
[16] L. M. Bollinger, R. E. Coté, R. T. Carpenter, and J. P. Marion, Phys. Rev. **132**, 1640 (1963).
[17] A. Bohr, Proc. Intern. Conf. Peaceful Uses At. Energy Geneva **2**, 151 (1956).
[18] L. Wilets and D. M. Chase, Phys. Rev. **103**, 1296 (1956).
[19] R. E. Lamphere, Nucl. Phys. **38**, 561 (1962).
[20] J. A. Northrop, R. H. Stokes, and K. Boyer, Phys. Rev. **115**, 1277 (1959).
[21] H. C. Britt, R. H. Stokes, W. R. Gibbs, and J. J. Griffin, Phys. Rev. Letters **11**, 343 (1963).
[22] J. J. Griffin and M. Rich, Bull. Am. Phys. Soc. **8**, 526 (1963).

In the comparison of the fission width distribution with a chi-squared distribution for a few degrees of freedom, there is an implicit assumption that the average partial fission width in each channel is the same size, i.e., that all the fission channels are open to the same extent. Wilets[12] has provided an approach which avoids this assumption, and which is probably much more realistic for the case of fission widths. Wilets defines an "effective" number of fission channels as the ratio

$$\nu_{\mathrm{eff}}=\frac{(\sum \nu_\alpha)^2}{(\sum \nu_\alpha^2)}=\frac{2\langle\Gamma_f\rangle^2}{\langle\Gamma_f^2\rangle-\langle\Gamma_f\rangle^2}, \qquad (1)$$

where $\langle\Gamma_f\rangle$ is the average fission width, and ν_α can be interpreted as the degree of openness of channel α. Wilets points out that Eq. (1) gives a value for ν_{eff} which is the same as the number of degrees of freedom of an equivalent chi-squared distribution, provided $\nu_\alpha=0$ or 1.

The estimate of the number of fission channels given by Eq. (1) is generally somewhat larger than that obtained from the approach of Wheeler,[23] who suggested that the number of channels should be related to the average fission width as

$$\nu_{\mathrm{total}}=\sum \nu_\alpha=2\pi\langle\Gamma_f\rangle/D, \qquad (2)$$

where D is the average spacing of the levels considered (i.e., those of a given spin and parity). Again, this estimate agrees with that given by Eq. (1) provided $\nu_\alpha=0$ or 1.

An estimate of the number of fission channels can also be obtained by considering interference effects among resonances in the fission cross section of fissile nuclei, using the method of Vogt.[24,25] Vogt defines an interference parameter $\cos\theta_{\lambda\lambda'}$, which gives the direction cosine between fission-width vectors of levels λ and λ' oriented in a multidimensional channel space. An average value of $|\cos\theta_{\lambda\lambda'}|$ of unity implies complete interference of all levels in a single-fission channel; an average value of zero implies no interference.

In the present analyses of the Pu^{241} fission cross section, it was found that complete interference could be assumed, i.e., the average interference parameter $|\cos\theta_{\lambda\lambda'}|$ for levels of the same spin is equal to unity. The data do indicate that there is a slight departure from complete interference, however, for the wide level group (group A in Table I). The narrow level group (group B) is not very sensitive to the presence of interference; it could possibly have been just as adequately treated under the assumption of no interference.

[23] J. A. Wheeler, Physica **22**, 1103 (1956).
[24] E. Vogt, Phys. Rev. **112**, 203 (1958).
[25] E. Vogt, Phys. Rev. **118**, 724 (1960).

B. Anomalies in Estimating the Number of Fission Channels

On the surface, it appears that estimating the number of fission channels in Pu^{241} by the various methods described above leads to contradictory results. Calculating the number of effective fission channels by the method of Wilets[12] with the multilevel parameters obtained for the resonances below 20 eV, one finds that there are 3.0 ± 0.6 channels for the wide-level group and 3.1 ± 0.9 for the narrow-level group. Estimating the number of fission channels according to the formula $\nu=2\pi\langle\Gamma_f\rangle/D$, as proposed by Wheeler,[23] one finds that there are 1.4 ± 0.2 channels in the wide-level group and 0.27 ± 0.10 in the narrow-level group.[26] From multilevel analysis, one finds that there is one (plus a small fraction) effective channel for the wide-level group. The narrow-level group is consistent with one or many channels, being indeterminate because the ratio of the average level width to the spacing is relatively small.

C. A Consistent View of the Number of Fission Channels

The fissile nuclide Pu^{241} is even-odd (as are the other common fissile nuclides U^{233}, U^{235}, and Pu^{239}), and the compound nucleus Pu^{242} is even-even. The spin and parity of Pu^{241} is $\frac{5}{2}^+$, so the compound nucleus spin states formed by s-wave neutron absorption are 2^+ and 3^+. According to the Bohr model it is assumed that the spectrum of fission channels for the excited fissioning compound nucleus Pu^{242*} is very similar to the spectrum of low-lying states near the ground state of the stable nucleus Pu^{242}. The spectrum of low-lying even-parity states in even-even heavy nuclei consists of a $K=0$ rotational band having the 0^+ ground state as its lowest member, and at an excitation of perhaps 1 MeV, quadrupole vibrational bands with $K=0$ and 2, as well as rotational bands built on intrinsic states with $K=0$. Each of these bands contains a member with $J=2$ but only the $K=2$ band contains a member with $J=3$. Experimentally, it appears[21,22] that the channel structure of even-even fissioning compound nuclides has about the same (or perhaps even larger) energy gap between the lowest $K=0$ band and the $K=2$ band. The fissile nuclide Pu^{241} is similar to U^{235} in that the fission threshold, presumably consisting of fission through the lowest ($K=0$) band of channels, occurs about 1 MeV below the energy afforded by slow neutron absorption.[27] Thus the $K=2$ channels and the higher $K=0$ channels may be very close to threshold at the excitation in Pu^{242} corresponding to Pu^{241} plus a zero-energy neutron. The crossing of the fission threshold is

[26] Since the reliability of the resonance parameters obtained from the analysis becomes more and more questionable at higher neutron energies, only those resonances below 20 eV were used in these calculations.
[27] See, for example, E. K. Hyde, U. S. Atomic Energy Commission Report No. UCRL-9036, 1962 Rev., p. 64 (unpublished).

not sharply defined in energy. From studies of fast-neutron-induced fission,[28–30] it appears that as much as 1 MeV may be required for a fission channel to open completely.

Consequently, for slow neutron fission of Pu^{241}, it may be reasonable to assume that the wide resonances are characteristic of channel spin 2^+, where there is one channel (with $K=0$) which is open to a relatively large extent, and several channels (with $K=0$ and 2) which are only open to a relatively small extent. The narrow levels, characteristic of channel spin 3^+, might be expected to consist of one or a few channels (with $K=2$ and perhaps 3), which are open to only a small extent.

It might be noted that Garrison,[14] who carried out a statistical analysis of fission widths of all four fissile nuclides, has remarked on the absence of resonances with very small fission widths. This is reasonable if it happens that there are several channels contributing to a very slight extent. Even though the probability of finding small widths in each slightly open channel is high (corresponding to a chi-squared distribution with 1 deg of freedom), the combined effect is to preclude small widths.

The fission widths obtained for Pu^{241} have been used in a qualitative calculation to check the consistency of these ideas. It was assumed that the widths of the narrow resonances in group B of Table I consist of channels which are open to about the same degree, so that $\sum_{\alpha=1}^{n} \nu_\alpha = n\langle\nu_\alpha\rangle$, where ν_α is the degree of openness of the channels and n is the number of channels. Since $n\langle\nu_\alpha\rangle \simeq 0.3$ and $(n\langle\nu_\alpha\rangle)^2/(n\langle\nu_\alpha\rangle^2) \simeq 3.1$ for the narrow resonances, $\langle\nu_\alpha\rangle \simeq 0.1$. For the wide resonances of group A, it was assumed that $\sum_{\beta=1}^{m+1} \nu_\beta = \nu_1 + \sum_{\alpha=1}^{m} \nu_\alpha = \nu_1 + m\langle\nu_\alpha\rangle$, where $\langle\nu_\alpha\rangle \simeq 0.10$. Since $\sum_\beta \nu_\beta \simeq 1.2$, and $(\sum_\beta \nu_\beta)^2/(\sum_\beta \nu_\beta^2) = 3.0$ for these resonances, it is found that $m \simeq 5$ and $\nu_1 \simeq 0.7$. From these numbers, it is possible to estimate the amount of interference which would be observed between two typical resonances. The magnitude of the interference parameter $|\cos\theta_{\lambda\lambda'}|$ turns out to be $\simeq 0.6$, which corresponds to a moderately large interference effect, so that in many cases an adequate fit to the data could be obtained by assuming complete interference among resonances, as was done in the multilevel analysis of the Pu^{241} fission data. Another test of this view of the fission-width distribution is the calculation of ν_{eff} for the single nearly-open channel, whose width distribution can be estimated to be that of the original distribution, less a constant for each level. The value of ν_{eff} for such a distribution is found to be 1.1 ± 0.4, agreeing with that which is expected from a single channel. One can conclude that in this way it is possible to remove (at least qualitatively) the apparent inconsistency of the results of various methods of estimating the number of fission channels.

The above considerations are based on the fission widths obtained from the present multilevel analysis of Pu^{241}. There is some evidence, however, that the behavior of the other even-odd fissile nuclides is quite similar. It has been previously noted[2] that the resonances of Pu^{241} are similar to those of U^{233}, in that two groups of resonances appear to exist with different properties. Recently an extensive multilevel analysis has been carried out by Kirpichnikov *et al.*[31,32] for resonances in U^{235} and Pu^{239}. Similar conclusions were reached: there appear to be two groups of resonances with somewhat different characteristics. In one group, the resonances appear to interfere almost completely, in a single-fission channel. In the other group, the resonances can be treated as noninterfering. The present considerations on Pu^{241} may thus be applicable to the resonance behavior of other fissile nuclides as well.

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